Chapters 16 and 17 cover convergence of sequences of operators, i.e., Korovkin-type theorems, and the representation of functions of several variables by superpositions of functions of fewer variables. Finally, there are four appendices, a large bibliography, and author and subject indexes.

This book clearly is of interest to anyone working in approximation theory.

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Gilbert G. Walter, *Wavelets and Other Orthogonal Systems with Applications*, CRC Press, Boca Raton, FL, 1994, x + 248 pp.

There has been an almost bewildering explosion in wavelet theory since the beginning of the 1990s. To the unwaved, such as the reviewer, the very large number of review articles and books that have appeared has left confusion as to where to start to learn this important subject. More specifically, as an academic with limited time, one wants to be able to pick up a book, to read it, and to incorporate parts of it into a course on approximation theory or harmonic analysis, or even to present a whole graduate course on the subject.

The book under review must surely be one of the best candidates for this purpose. There are many very readable accounts of the theory, starting from different viewpoints, such as applications to signal processing or an account of the analysis. Many start with an account of the relation to Fourier transforms and series, or to general orthogonal series. In this book, the author has integrated the theory of general orthogonal systems, and especially orthogonal polynomials with wavelet theory, to a far greater degree. In that sense, it is less specialized than most of the other books and easier for those (such as the reviewer) that come from a background of orthogonal polynomials or classical analysis.

The author notes that his target audience consists of engineering and mathematics graduate students, and that large parts of the book have been tested on a mixed class of engineering and graduate students. He also notes that the book could be used for courses on special functions or signal processing. Certainly the attractive print and style of presentation, the numerous examples given, and the introductions given to topics such as tempered distributions, sampling theorems, and statistical applications make even individual chapters attractive for selection for diverse courses. There is also a “nice light feel” to individual sections: ideas are presented without clobbering the reader with too much technical detail.

Chapter 1 contains an introduction to orthogonal functions and series, with specific attention to the trigonometric/Fourier system, and to the Shannon and Haar systems as precursors of wavelets. Chapter 2 contains the basic theory of tempered distributions. In Chapter 3, the author begins the discussion of wavelets, with multiresolution analysis, and mother wavelets, followed by several examples. Mallat’s decomposition and reconstruction algorithm is presented, as well as an application to filters.

In Chapter 4, some of the classical convergence theorems for Fourier series are presented, together with less familiar ones for Fourier series of periodic distributions. Abel summability and Fejér means are also briefly discussed. In Chapter 5, the relationship between wavelets and distributions is investigated, involving topics such as wavelets based on distributions or multiresolution analysis of distributions.

In Chapter 6, classical orthogonal polynomials (Jacobi, Hermite, and others) are discussed, while orthogonal systems generated by Sturm–Liouville equations and the Walsh/Rademacher systems are presented in Chapter 7. The latter also treats orthogonal and biorthogonal wavelets. In Chapter 8, general so-called “delta sequences” are introduced, the classic examples of course being suitably scaled convolution kernels. This is applied to pointwise convergence of distribution expansions. Rates of convergence are also discussed, as are Gibbs phenomena and general partial sums of wavelet expansions. Shannon sampling theorems are presented in Chapter 9, with many different types of sampling.

Chapter 10 is of special interest, as it explores the extent to which translation invariance extends from classical Fourier series/transforms to other orthogonal systems and to what extent there is dilation invariance. The comparison of wavelet, Fourier, and orthogonal polynomial systems, which is one of the central themes of this text, is especially present here. It is shown how specific wavelets can exhibit very simple transformations with respect to translation/dilation, while others have none at all! Weak forms of invariance are also discussed.

In Chapter 11, analytic representations via orthogonal series are discussed, while statistical applications to density estimation and stochastic processes are given in the final two chapters.

Each chapter ends with a problem section: the choice of problems seems quite reasonable and within the range of good graduate students. The bibliography offers students selected references for more detailed study.

This is a most attractive book for mathematicians wishing to learn the basics of wavelet theory, in small doses, and with perspectives given to compare them to more familiar objects. It is also very well suited for its main aim, as a textbook for beginning graduate/senior undergraduate courses on orthogonal systems, harmonic analysis, special functions, and some of their applications.

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Guido Walz, *Asymptotics and Extrapolation*, Mathematical Research **88**, Akademie Verlag, Berlin, 1996, 330 pp.

In numerical analysis and in applied mathematics, many methods produce sequences converging to the solution of the problem. This is the case of iterative methods of various origins and of methods producing a result depending on a parameter (for example, the step size) and tending to the exact solution when the parameter tends to zero. This typical situation arises, for example, in quadrature methods, in discretization methods for ordinary and partial differential equations, and in the approximation of functions.

In many cases, these sequences converge very slowly and it is mandatory to transform them into new sequences converging faster to the same limit. Such transformations are usually called convergence acceleration methods or extrapolation methods. They consist in transforming the slow sequence into another one without modifying the construction of the sequence to be accelerated. The most famous example is Romberg's method for accelerating the trapezoidal rule. Another well-known convergence acceleration method, which works on all linearly converging sequences, is Aitken's Δ^2-process. There exist many extrapolation methods in the literature and the choice of one of them for accelerating the convergence of a given sequence is, of course, based on some theoretical considerations.

It was proved some years ago that, in order to accelerate the convergence of a sequence (x_n) converging to a limit x when n tends to infinity, it is necessary to know an asymptotic expansion of the error $x_n - x$. So, asymptotics and extrapolation methods are two subjects which are, by nature, closely related.

Let us mention that extrapolation methods also lead to new algorithms for the solution of various problems. For example, everyone knows Steffensen's method which converges quadratically for computing fixed points and which is based on Aitken's Δ^2-process. Moreover, convergence acceleration methods are related to many other important topics such as Padé approximation, continued fractions, formal orthogonal polynomials, and projection methods